Plan: T 2.1, 2.2 | T 3.2, 3.3 | T off | Due Tues
Th 2.3, 3.1 | Th Exam 2 | Th 4.1, 4.2

1.10 – Applications of Linear Systems

We will consider 3 types of applications:

- Electrical circuits w/ batteries & resistors
- Balancing chemical reactions
- Polynomial interpolation

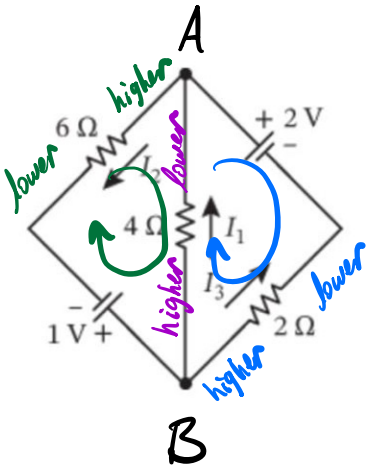
Circuits: → symbol +|+ symbol —∿∿—

A **battery** is a source of electric energy, and a **resistor** dissipates electric energy. A **node** is where three or more wires join in a circuit. A **closed loop** begins and ends at the same node.

Ohm's Law: $E = IR$, where E (volts) is voltage drop at a resistor with resistance R (ohms) in a circuit with current I (amperes).

Kirchhoff's Laws (summary): Net current (in and out) at a node is zero, and net voltage change (rises and drops) in a closed loop is zero.

6. Analyze the given electrical circuit by finding the unknown currents.



Node A: $I_1 + I_3 - I_2 = 0$

Node B: $I_2 - I_1 - I_3 = 0$

equivalent

clockwise right loop:

voltage raises	voltage drops
$2I_3$	$-2 - 4I_1 = 0$

CW left loop:

$6I_2 + 4I_1$	$- 1 = 0$

System: $I_1 - I_2 + I_3 = 0$

$-4I_1 + 2I_3 = 2$

$4I_1 + 6I_2 = 1$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ -4 & 0 & 2 & 2 \\ 4 & 6 & 0 & 1 \end{array}\right]$$

We find that

$I_1 = -5/22$ A

$I_2 = 7/22$ A

$I_3 = 6/11$ A

10. Write a balanced equation for the given chemical reaction.

$x\ C_6H_{12}O_6 \rightarrow yCO_2 + zC_2H_5OH$ [fermentation of sugar]

C: $6x = y + 2z$ $\quad 6x - y - 2z = 0$

H: $12x = 6z$ $\quad \rightarrow \quad 2x - z = 0$

O: $6x = 2y + z$ $\quad 6x - 2y - z = 0$

$$\begin{bmatrix} 6 & -1 & -2 \\ 2 & 0 & -1 \\ 6 & -2 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -1/2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{matrix} \Rightarrow x = \frac{1}{2}z \\ \Rightarrow y = z \end{matrix}$$

Let $t = z$. Then $x = \frac{1}{2}t$, $y = t$, $z = t$

Let $t = 2$. Then $x = 1$, $y = 2$, $z = 2$

$x\ C_6H_{12}O_6 \rightarrow yCO_2 + zC_2H_5OH$

$$C_6H_{12}O_6 \rightarrow 2CO_2 + 2C_2H_5OH$$ ✓

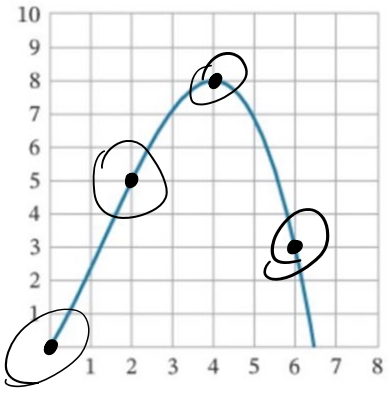
Theorem 1.10.1 Polynomial Interpolation

Given ~~any~~ n points in the xy-plane that have distinct x-coordinates, there is a unique polynomial of degree $n-1$ or less whose graph passes through those points.

Need 4 points

degree: 3

16. The accompanying figure shows the graph of a cubic polynomial. Find the polynomial.



$ax^3+bx^2+cx+d=y$

(2,5): $8a+4b+2c+d=5$

(4,8): $64a+16b+4c+d=8$

(6,3): $216a+36b+6c+d=3$

(0,0): $d=0$

$$\left[\begin{array}{ccc|c} 8 & 4 & 2 & 5 \\ 64 & 16 & 4 & 8 \\ 216 & 36 & 6 & 3 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1/8 \\ 0 & 1 & 0 & 1/2 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

The polynomial is $y=-\frac{1}{8}x^3+\frac{1}{2}x^2+2x$.